MOTION OF THE SOLID PHASE OF AN AEROSOL AT LARGE REYNOLDS NUMBERS

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Improved singly linked expressions are presented for the drag coefficient of the carrying agent and the solution of the equations of motion of the solid phase of an aerosol at large Reynolds numbers.

The investigation of the motion of the solid phase in streams of gas containing small dust concentrations can be reduced to the study of motion under the action of various external forces on individual particles in a resisting medium. The analysis of the motion of the solid particles of an aerosol is usually based on the force-balance equation. For the nonuniform motion of a particle of constant mass this equation can be written in the form

$$m\frac{d\mathbf{V}}{dt}=\Sigma\,\mathbf{F}'+\mathbf{F}. \tag{1}$$

The solution of Eq. (1) is largely dependent on the expression taken for the resisting force of the carrying agent of the aerosol.

For rectilinear motion the resistance offered by a gaseous medium to the motion of spherical bodies (dust particles) is given by the generally accepted expression

$$F=\psi\,\frac{\pi d^2}{4}\cdot\frac{W^2}{2}\,\rho. \tag{2}$$

The use of multiply linked relations to calculate the drag coefficient appearing in this expression complicates the theoretical investigation, since in this case the expression for the resisting force consists of two and more terms. Singly linked expressions of the type

$$\psi=\frac{A}{\mathrm{Re}^n} \tag{3}$$

are more convenient. In a number of cases, however, the known equations of this type give an error of considerably more than 10%. A study of the experimental values of the drag coefficient for spheres given in [1] showed that for Reynolds numbers of interest for dust precipitation (Re < 1000) the dependence of the drag coefficient on Re can be accurately described over discrete intervals by the equation of a straight line

$$\lg\psi=\lg A-n\lg\mathrm{Re}, \tag{4}$$

which corresponds to Eq. (3). Table 1 shows the values found for A and n in Eqs. (3) and (4), and the greatest deviation of the values calculated by these equations from the experimental data.

The substitution of Eq. (3) into the expression for the resisting force of a gaseous medium (2) leads to a relation which is convenient for theoretical investigation

$$F=0.125\,A\pi\rho\nu^n d^m W^m, \tag{5}$$

where

$$m=2-n. \tag{6}$$

For Re < 0.1 (A = 24, n = 1) the general problem (5) goes over into Stokes's law.

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TABLE 1. Values of A and n in Eqs. (3) and (4)

Re	A	n	Deviation from experimental value of ψ, %
<0,1*)	24	1,0	−1,5
0,1—1,0	26,9	0,95	±2
1,0—10	26,5	0,8	∓3,5
10—100	16,8	0,6	∓3
100—800	6,1	0,38	∓3,2
100—1000	5,8	0,37	±4

*Stokes's law.

The common structure of singly linked power laws for determining the drag coefficient (3) enables us to obtain solutions in general form, and for a numerical solution of the problem it enables us to substitute the appropriate values of A and n from Table 1 into the general relations obtained.

For nonspherical particles Eq. (5) takes the form

$$F = 0.125\, kA\pi\rho\nu^n d_e^m W^m. \tag{7}$$

Substituting Eq. (7) into (1) gives an equation for the rectilinear motion of particles of arbitrary shape which is applicable over a wide range of Reynolds numbers;

$$\frac{dV}{dt} = X - \frac{W^m}{\tau}, \tag{8}$$

where

$$\tau = \frac{d_e^{n+1}\rho_1}{0.75kA\rho\nu^n}. \tag{9}$$

When the velocity of the carrying agent of the aerosol is constant dW = dV and Eq. (8) can be reduced to the form

$$\frac{dW}{X\tau - W^m} = \frac{dt}{\tau}. \tag{10}$$

The general solution of Eq. (10) for constant X can be obtained by expanding the denominator of the left-hand side in a binomial series. If W = 0 at t = 0, the solution of Eq. (10) is

$$t = \frac{W}{X}\left(1 + \frac{\alpha}{m+1} + \frac{\alpha^2}{2m+1} + \frac{\alpha^3}{3m+1} + \frac{\alpha^4}{4m+1} \dots\right), \tag{11}$$

where

$$\alpha = \frac{W^m}{a}, \tag{12}$$

$$a = X\tau. \tag{13}$$

The binomial series in Eq. (11) converges for $-1 < \alpha < 1$. The value of m depends on how the carrying agent of the aerosol flows around a particle (Re) and is determined as a function of the exponent n in the equation for the drag coefficient (3) (Table 1). As n varies from 1 to 0, m varies from 1 to 2. Fuchs [1] obtained a particular solution of Eq. (8) for m = 1 (n = 1). The solution for m = 2 (n = 0) which corresponds to Re > 1000 is shown below. Equation (10) is written in the form

$$\frac{dW}{a_1^2 - W^2} = \frac{dt}{\tau}, \tag{14}$$

where

$$a_1 = \sqrt{a}. \tag{15}$$

The integration of Eq. (14) [2] and routine transformations lead to the solution

$$W = a_1 \frac{\beta \exp\left(2a_1 \frac{t}{\tau}\right) - 1}{\beta \exp\left(2a_1 \frac{t}{\tau}\right) + 1}. \tag{16}$$

Here

$$\beta = \frac{1 + \alpha_1}{1 - \alpha_1}, \tag{17}$$

$$\alpha_1 = \frac{W}{a_1} < 1. \tag{18}$$

If the particle velocity W = 0 at t = 0, α_1 = 0 and β = 1. For these initial conditions

$$W = a_1 \frac{\exp\left(2a_1 \frac{t}{\tau}\right) - 1}{\exp\left(2a_1 \frac{t}{\tau}\right) + 1}. \tag{19}$$

Solutions (16) and (19) correspond to the initial conditions assumed. It follows from (16) that for t = 0 the relative velocity W = $\alpha_1 a_1$, and from (19) W = 0.

In both cases W → a_1 as t → ∞.

The steady precipitation of particles under the action of gravity in an ascending stream of gas is of great practical interest. This process occurs, for example, in gravitational (centrifugal) dust precipitators. In this case (dV/dt = 0, X = g) Eq. (8) takes the form

$$g - \frac{W_c^m}{\tau} = 0. \tag{20}$$

This equation leads to an expression for calculating the steady precipitation velocity (rate of settling) of particles for large Reynolds numbers

$$W_c = (g\tau)^{\frac{1}{m}} = \left[\frac{g\rho_1 d_e^{n+1}}{0.75\, kA\rho\nu^n}\right]^{\frac{1}{m}}. \tag{21}$$

For the precipitation of spherical particles (k = 1) for Re < 1 (A = 24, n = 1) Eq. (21) takes the familiar form

$$W_c = \frac{g\rho_1 d^2}{18\mu}. \tag{22}$$

It follows from (21) that the coefficient a_1 (15) in Eqs. (14), (16), and (19) is just the rate of settling of particles for a quadratic resistance law (m = 2).

In treating the precipitation of dust particles under the influence of gravity in an ascending nonisothermal gas flow it is assumed that the temperature along the channel varies enough so that the difference between the temperature of the particles and that of the gas is negligible. We use the procedure of [3] and assume the relations

$$U = U_0 \frac{T}{T_0}, \quad \rho = \rho_0 \frac{T_0}{T}, \quad \mu = \mu_0 \left(\frac{T}{T_0}\right)^{n_1} \text{and } T = T_0 - bx. \tag{23}$$

Substituting the expressions for the viscosity and density of the gas (23) into (21) we find a relation for the rate of settling of the particles

$$W_c = \left[\frac{g\rho_1 d_e^{n+1}}{0.75kA\rho_0\nu^n}\right]^{\frac{1}{m}} \left(\frac{T_0}{T}\right)^{\beta_1}, \tag{24}$$

where

$$\beta_1 = \frac{n(n_1+1)-1}{m}. \tag{25}$$

In the special case of the precipitation of spherical particles for Re < 0.1 (k = 1, A = 24, n = 1) Eq. (24) takes the form

$$W_c = \frac{g\rho_1 d^2}{18\mu_0}\left(\frac{T_0}{T}\right)^{n_1}. \tag{26}$$

In Eqs. (24) and (26) the values of the temperature are calculated from (23). When the solid phase and the carrying agent of the aerosol have the same temperature [3]

$$b = \frac{\pi D\alpha_T \Delta T}{\omega}. \tag{27}$$

Since n and m depend on the values of the Reynolds number, the exponent β_1 in (25) is a function of both Re and n_1. The value of β_1 decreases with increasing Reynolds number; i.e., the decrease in viscosity resulting from a lowering of the temperature of the gas has a smaller effect on the rate of settling.

Fuchs [1] presents the expression

$$\mathbf{F} = k\psi\rho\frac{\pi d_e^2}{8}(\mathbf{V}-\mathbf{U})|(\mathbf{V}-\mathbf{U})| \tag{28}$$

for the resisting force of the carrying agent of an aerosol for curvilinear motion of solid particles at Re > 0.5.

This equation contains the absolute value of the relative velocity of the particles which determines the characteristics of the motion along the various coordinate axes.

If Eq. (3) is used for the drag coefficient and, in accord with [1], the expression

$$\mathrm{Re} = \frac{d_e|(\mathbf{V}-\mathbf{U})|}{\nu} \tag{29}$$

is substituted into it and some transformations performed, the expression for the resisting force of the carrying agent (28) is reduced to the form

$$\mathbf{F} = 0.125\,\pi kA\rho\nu^n(\mathbf{V}-\mathbf{U})|(\mathbf{V}-\mathbf{U})|^{1-n} d_e^m. \tag{30}$$

The substitution of Eq. (30) into (1) gives an equation for the curvilinear motion of a particle of arbitrary shape which is applicable over a wide range of Reynolds numbers:

$$\frac{d\mathbf{V}}{dt} = X - B(\mathbf{V}-\mathbf{U})|(\mathbf{V}-\mathbf{U})|^{m_1}, \tag{31}$$

where

$$B = 0.75\,kA\nu^n\frac{\rho}{\rho_1 d_e^{n+1}}, \tag{32}$$

$$m_1 = 1 - n. \tag{33}$$

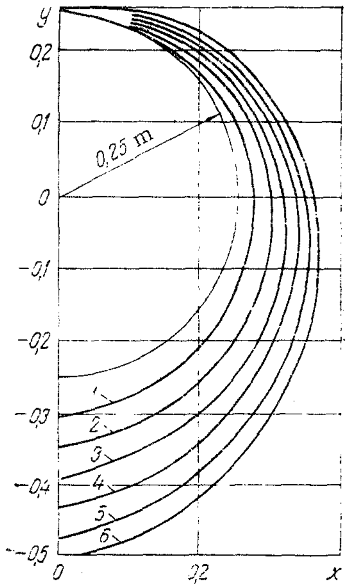

Fig. 1. Trajectories of 20-μ dust particles as a function of density: 1) ρ_1 = 1000 kg/m^3; 2) 2000; 3) 3000; 4) 4000; 5) 5000; 6) 6000; x and y in m.

Equation (31) was solved for the steady motion of an aerosol in a channel of constant cross section and constant curvature in a vertical plane. Particles suspended in the gas entering the curved channel from a rectilinear gas conduit tend by inertia to maintain their rectilinear motion. The trajectories of these particles are curved as a result of a number of forces acting on them, the most effective being the interaction (resistance) of the carrying agent and gravity.

On the basis of studies [4] showing that the actual field of tangential velocities of the carrying agent has practically the same effect on the trajectories of the particles of an aerosol as a uniform field, the velocity of the gas is taken at all points equal to the average velocity and directed along the tangents to circles described about the center of curvature of the channel which is taken as the origin of coordinates.

In this case

$$U_x = U \frac{y}{\sqrt{x^2 + y^2}}, \tag{34}$$

$$U_y = U \frac{x}{\sqrt{x^2 + y^2}}. \tag{35}$$

Using the fact that

$$|(\mathbf{V} - \mathbf{U})| = [(V_x - U_x)^2 + (V_y - U_y)^2]^{\frac{1}{2}},$$

and replacing V_x and V_y by dx/dt and dy/dt, respectively, we obtain the equations

$$\begin{aligned} \frac{d^2x}{dt^2} &= -B\left(\frac{dx}{dt} - U\frac{y}{\sqrt{x^2+y^2}}\right)\left[\left(\frac{dx}{dt} - U\frac{y}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{dy}{dt} - U\frac{x}{\sqrt{x^2+y^2}}\right)^2\right]^{\frac{m_1}{2}}, \\ \frac{d^2y}{dt^2} &= -B\left(\frac{dy}{dt} - U\frac{x}{\sqrt{x^2+y^2}}\right)\left[\left(\frac{dx}{dt} - U\frac{y}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{dy}{dt} - U\frac{x}{\sqrt{x^2+y^2}}\right)^2\right]^{\frac{m_1}{2}} - cg. \end{aligned} \tag{36}$$

The solution of system (36) was obtained by computer and used to calculate the trajectories of dust particles 10-100 μ in diameter having densities of 1000-6000 kg/m^3. Figure 1 shows the trajectories of 20-μ dust particles in a curved channel of radius 0.25 m for a 25 m/sec velocity of the carrying agent (air). These data were used to develop an experimental inertial dust separator with a capacity of 1500 m^3/h.

NOTATION

$\Sigma P'$, sum of external forces acting on a particle which are not related to its displacement; F, resisting force of carrying agent of aerosol; m, ρ_1, d, V, and W, mass, density, diameter, absolute and relative velocities of particle; d_e, equivalent diameter of particle; t, time; U, ρ, μ, ν, velocity, density, absolute and kinematic viscosities of carrying agent of aerosol; $\pi = 3.14$, ratio of circumference to diameter; ψ, drag coefficient of spherical dust particles; $Re = Wd/\nu$, Reynolds number for particle; A and n, coefficients (Table 1); k, shape factor of particle; $X = \Sigma F'/m$, acceleration produced by forces on particle which are not related to its displacement; e, base of natural logarithms; W_c, rate of settling of particles; T, absolute temperature; n_1, exponent; b, coefficient depending on rate of heat exchange of gas with channel walls and suspended particles; 0, subscript indicating that quantity refers to conditions at entrance to section under consideration; D, diameter of apparatus; α_T, heat-transfer coefficient; ΔT, temperature head; ω, water equivalent of flow; x, y, running coordinates; g, acceleration due to gravity; $c = 1 - \rho/\rho_1$, coefficient taking account of buoyancy of gas.

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GRAVITATIONAL SEPARATION OF DISPERSE SYSTEMS WITH PRELIMINARY AGGREGATION OF PARTICLES IN AN ELECTRIC FIELD

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The separation of a disperse medium in a gravitational sedimentation tank is investigated. An external electric field is used for preliminary aggregation of the conducting disperse phase.

At present, the separation of disperse media (for example, the removal of oil from water) is carried out by gravitational precipitation of the disperse-phase particles in a sedimentation apparatus, with preliminary aggregation of the particles. If the particles are conducting, more intense aggregation (coalescence) will occur when an external electric field is applied. A coalescence unit is an apparatus for the preliminary aggregation of emulsions in a constant electric field. In spite of the wide use of sedimentation apparatus, there has been little study of its efficiency.

All the existing types of sedimentation tanks may be divided into two classes: those with vertical and horizontal fluxes in the sedimentation region. One of the main characteristics of these units determining the efficiency of separation of disperse media is the ratio between the concentrations of the disperse phase at the outlet and inlet of the sedimentation tank

$$\lambda = \frac{W_2}{W_1}. \tag{1}$$

It is possible to establish which of the two types of sedimentation tanks is the more efficient for the separation of disperse media from a comparison of the respective values of λ. The more efficient apparatus will have the lower value of λ.

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